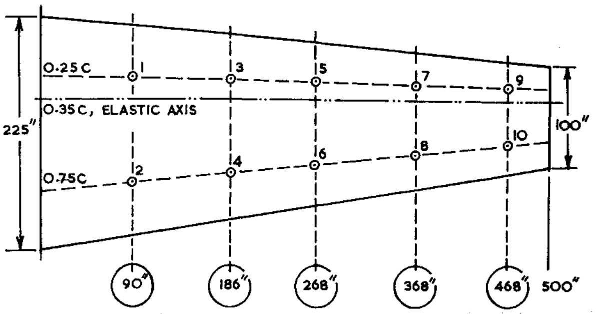

Fig. 3 Jet transport wing.

Using strip theory, it can be shown that the matrix of aerodynamic influence coefficients is given by

$$[C_h] = (S/(e+d))\begin{bmatrix} 1 & 1/(e+d) \\ 0 & -1/(e+d) \end{bmatrix}\begin{bmatrix} 0 & C_{L\alpha} \\ 0 & c\cdot C_{m\alpha} \end{bmatrix}\begin{bmatrix} d & e \\ 1 & -1 \end{bmatrix} \tag{6}$$

where S is the plan area of the model and c is its chord.

Equations (5) and (6), when combined, give the eigenvalue problem (Eq. 4) which is then solved to obtain the value of q_D. Since the lift and the moment are referred to the aerodynamic centre, $C_{m\alpha} = 0$ and the eigenvalues are

$$(1/q_D) = 0 \qquad S\cdot e\cdot C^{\alpha}\cdot C_{L\alpha} \tag{7}$$

The zero root corresponds to a pure translation mode ($h_1 = h_2$). The second root shows, as expected, that q_D is independent of C^{α} and is the result obtained from a consideration of moments about the elastic axis.[1]

2) As a second example, the symmetrical divergence speed of a hypothetical jet transport wing, analysed by Bisplinghoff et al. in their book,[1] will be calculated by the present method. The wing is divided into five spanwise stations (Fig. 3) with control points located on the quarter-chord and the three-quarter-chord lines. The flexibility matrix for this system of control points has been calculated by Rodden[2] from data given in Ref. 1. Strip theory aerodynamics was used to set up the matrix of aerodynamic influence coefficients. (The value of $C_{L\alpha}$ was taken as 3.3325/rad.) The results of the analyses using 1, 2, 3, and 5 spanwise stations are shown in Table 1 which also shows the results obtained in Ref. 1 using conventional methods.

One problem encountered in solving Eq. (4) for this particular problem was the following: with the present choice of two control points at each spanwise control station, and with the use of strip theory aerodynamics, Eq. (4) will have as many zero roots as there are spanwise stations. (These correspond to pure bending modes of the wing.) Eq. (4) can be rearranged, in partitioned form, as

$$\left[\begin{array}{c|c} [A] - (1/q_D)[I] & -[A] \\ \hline [B] & -[B] - (1/q_D)[I] \end{array}\right]\begin{Bmatrix} h_f \\ \hline h_r \end{Bmatrix} = \begin{Bmatrix} 0 \\ \hline 0 \end{Bmatrix} \tag{8}$$

where the subscript f refers to the control points on the 0.25c line and r refers to those on the 0.75c line. The matrices $[A]$ and $[B]$ are of order ($N \times N$) where $2N$ is the total number of control points. The zero roots can be eliminated from Eq. (8) and an eigenvalue problem for the nonzero roots can be obtained as

$$(1/q_D)\{h_r\} = [B]([A][B]^{-1} - [I])\{h_r\} \tag{9}$$

The values of $\{h_f\}$ corresponding to a particular eigenvalue can be obtained from

$$[I]\{h_f\} = ([I] + (1/q_D)[B]^{-1})\ \{h_r\} \tag{10}$$

Discussion of the Results and Conclusions

It can be seen from Table 1 that even with three spanwise stations, the divergence speed obtained from the present method is of comparable accuracy to that obtained in Ref. 1 using strip theory. The results of the 5 station analysis using lifting line theory is comparable to the corresponding result of Ref. 1. (Since the aim of these calculations was only to obtain an estimate of the accuracy of the present method, no corrections for compressibility have been applied.) Since no prior assumptions need to be made, this method is applicable to a wing of general planform to predict both the torsional and the chordwise divergence speeds.

References

[1] Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison-Wesley Publishing Co., Cambridge, Mass., Chap. 8, 1955, pp. 427–440.

[2] Rodden, W. P., "A Matrix Approach to Flutter Analysis," SMF Fund Paper FF-23, Institute of the Aeronautical Sciences, New York, N.Y. May 1959.

Subsonic Similarity Rule for Jet-Flapped Airfoil

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Introduction

A SOLUTION of the two dimensional, jet-flapped, symmetric wing in compressible flow by the methods of thin-airfoil theory has been reported in Ref. 1. The analysis assumes that 1) the flow inside the jet is irrotational and bounded by vortex sheets across which it is prevented from mixing with the main stream and 2) the jet is infinitely thin, but possesses finite momentum. It is the aim of this report to extend the analysis, by means of similarity transformations, to the case of a two-dimensional, jet-flapped, symmetric wing in subsonic flow.

Outline of Incompressible Solution

A jet issues from the trailing edge of an airfoil, at angle of attack α_1, and enters an incompressible stream with a deflection τ_1, Fig. 1, where U_1 denotes the freestream velocity, c the chord length, and y_{j_1} the jet shape. In terms of the perturbation velocity potential ϕ the linearized flow equation is

$$\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 = 0 \tag{1}$$

The boundary conditions are mixed. On the wing

$$(\partial\phi/\partial y)_{y=0} = U_1\alpha_1 \qquad 0 < x < c \tag{2}$$

whereas on the jet

$$(\partial\phi/\partial y)_{y=0} = U_1 y_{j_1}'(x) \qquad c < x < \infty \tag{3}$$

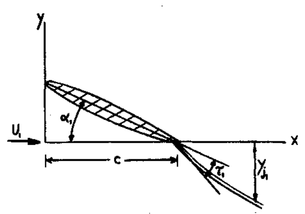

Fig. 1 Jet flap configuration.

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The deflection of the jet at the trailing edge is

$$y_{j_1}'(c) = \alpha_1 + \tau_1 \quad (4)$$

and at infinity

$$y_{j_1}'(\infty) = 0 \quad (5)$$

The solution of Eqs. (1–5), Ref. (1), gives an expression for the derivative of the jet shape of the form

$$y_{j_1}'(x) = \alpha_1 F(x,C_{j_1}) + \tau_1 G(x,C_{j_1}) \quad (6)$$

where C_{j_1} is the jet momentum coefficient defined as the jet momentum per unit span divided by the product of the wing chord and freestream dynamic pressure. The lift coefficient may be written

$$C_{L_1} = \tau_1 \partial C_{L_1}/\partial \tau_1 + \alpha_1 \partial C_{L_1}/\partial \alpha_1 \quad (7)$$

where for $0 < C_{j_1} < 10$ the following formulas for the lift derivatives are applicable

$$\partial C_{L_1}/\partial \tau_1 = 2(\pi C_{j_1})^{1/2}(1 + 0.151C_{j_1}^{1/2} + 0.139C_{j_1})^{1/2} \quad (8)$$

$$\partial C_{L_1}/\partial \alpha_1 = 2\pi(1 + 0.151C_{j_1}^{1/2} + 0.219C_{j_1}) \quad (9)$$

Formulation of Similarity Rule

Following Ref. 2, consider the potential function $\Phi(\xi,\eta)$ of a compressible flow in (ξ,η) coordinates having freestream Mach number M and freestream velocity U_2. Choose Φ to be related to ϕ by

$$\phi(x,y) = A(U_1/U_2)\Phi(\xi,\eta) \quad (10)$$

$$\xi = x,\ \eta = y/(1 - M^2)^{1/2} \quad (11)$$

where A is a constant. Introducing Eqs. (10) and (11) into Eq. (1) yields

$$\partial^2\Phi/\partial\xi^2 + [1/(1 - M^2)]\partial^2\Phi/\partial\eta^2 = 0 \quad (12)$$

Hence Φ is a solution corresponding to Mach number M. The boundary conditions (2) and (3) yield

$$(\partial\Phi/\partial\eta)_{\eta=0} = U_2\alpha_2 \qquad 0 < \xi < c \quad (13)$$

$$(\partial\Phi/\partial\eta)_{\eta=0} = U_2[\alpha_2 F(\xi,BC_{j_2}) + \tau_2 G(\xi,BC_{j_2})] \qquad c < \xi < \infty \quad (14)$$

provided

$$\alpha_1(1 - M^2)^{1/2}/A = \alpha_2, \quad \tau_1(1 - M^2)^{1/2}/A = \tau_2, \quad C_{j_1} = BC_{j_2} \quad (15)$$

It is shown below that B can be chosen so as to make the bracketed term of Eq. (14) equal to the derivative of the jet shape in the (ξ,η) plane. Thus, Eq. (14) becomes

$$(\partial\Phi/\partial\eta)_{\eta=0} = U_2 y_{j_2}'(\xi) \qquad c < \xi < \infty \quad (16)$$

Equations (4) and (5) are similarly transformed.

The pressure coefficient of the incompressible solution may be related to the pressure coefficient of the compressible solution by

$$C_{p_1} = -(2/U_1)(\partial\phi/\partial x)_{y=0} = -(2A/U_2)(\partial\Phi/\partial\xi)_{\eta=0} = AC_{p_2} \quad (17)$$

A normal momentum balance about the jet yields the following relationship between the pressure discontinuity across the jet and its radius of curvature

$$\Delta C_p = -cC_j/R \quad (18)$$

where R is the radius of curvature. Thus the radius of curvature of the jet shapes in the two flow systems are related by

$$R_2 = -cC_{j_2}/\Delta C_{p_2} = -cC_{j_1}A/B\Delta C_{p_1} = AR_1/B \quad (19)$$

It is easily demonstrated, within the approximations of thin-airfoil theory, that

$$1/R_1 = -d^2y_{j_1}/dx^2,\ 1/R_2 = -d^2y_{j_2}/d\xi^2 \quad (20)$$

Combine the two preceding expressions withEqs. (6, 15, and 19) to obtain

$$y_{j_2}'(\xi) = [B/(1 - M^2)^{1/2}][\alpha_2 F(\xi,BC_{j_2}) + \tau_2 G(\xi,BC_{j_2})] \quad (21)$$

Imposing the condition $B = (1 - M^2)^{1/2}$ verifies Eq. (16). Owing to the homogeneity of the differential equations the constant A remains arbitrary. The similarity law expressed by Eqs. (15) and (17) may be written in the general form

$$C_p/A = f[\alpha/A(1 - M^2)^{1/2},\ \tau/A(1 - M^2)^{1/2},\ C_j(1 - M^2)^{1/2}] \quad (22)$$

Choosing $A = 1$ the incompressible solution can be generalized to a subsonic flow by replacing α_1, τ_1, and C_{j_1} appearing in the incompressible expressions by $\alpha/(1 - M^2)^{1/2}$, $\tau/(1 - M^2)^{1/2}$, and $C_j(1 - M^2)^{1/2}$, respectively.

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[1] Spence, D. A., "The Lift Coefficient of a Thin, Jet-Falpped Wing," *Proceedings of the Royal Society of London*, Ser. A, Vol. 238, No. 1212, 1956, pp. 46–68.

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Downwash Correction for a Two-Dimensional Finite Wing

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A FINITE two-dimensional wing in a wind tunnel behaves as a three-dimensional wing because of the downwash induced by tip and wall vortices. For design work the true two-dimensional flow over the wing section is desired and much effort is usually expended in test programs to correct for the downwash. In the discussion to follow it is demonstrated that the true two-dimensional flow over the two-dimensional finite wing can be found without the need to correct, experimentally, for the downwash provided the pressure distribution over the section is known from test data.

As an illustration of the method consider the cambered ellipse wing section shown in Fig. 1. Its wing span was 16.5 in. and it was tested in the West Virginia University subsonic tunnel at a geometric angle of attack $\alpha_g = 10°$ and a freestream velocity $V_\infty = 150$ fps. The cambered ellipse section had a thickness-chord ratio of 20%, a 5% camber, and a chord $c = 9$ in.; 37 pressure taps were arranged around its periphery at the wing midspan. Vectored blowing was introduced at the blowing slot and the pressure distribution shown by the open encircled points in Fig. 1 was obtained.[1] This test was part of a continuing program to study the feasibility of obtaining high-lift devices by vectored blowing around bluff-ended bodies.[2,3]

From the test data the normal sectional coefficient c_n and the chordwise sectional coefficient c_h are obtained from the

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